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Role of turbulence on the radiation from radiating sources in plasmas

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Abstract. The role of turbulence on the radiation pattern of a radiating source (antenna) is discussed. A partial analysis of the radiation from an electric dipole in a turbulent plasma medium is made and an expression for the total energy flux of the dispersed field is obtained. The energy flux of the dispersed field is calculated for the plasma parameters in different situations.

1. Introduction

When the electromagnetic fields of electromagnetic energy sources exist in a plasma, non-uniform regions or sheaths will develop between the material boundaries of these sources and the plasma. Furthermore, because of the thermal properties of the plasma, acoustic disturbances may also be generated. Thus a marked interaction between a source of electromagnetic energy located in a plasma and the plasma can develop. A variety of antenna configurations have been analysed by several authors (Mittra and Deschamps 1963, Seshadri 1965, Seshadri and Yip 1966, Wait 1964) to obtain tractable solutions to simplified boundary conditions.

Electric dipoles in plasmas have received considerable attention. In general, the radiation resistance is reduced by temperature when the frequency of radiation is less than the plasma frequency. For arbitrary orientation of the dipole to the direction of magnetic field, Mittra and Deschamps (1963) arrived at the far-field pattern in terms of a finite-range integral suitable for numerical computation. Seshadri (1965) and Weil and Walsh (1964) have evaluated the radiation resistance; calculations have also been made on electric dipole antennae for operation in the ionosphere (Seshadri and Yip 1966). Because of its two-dimensional symmetry the magnetic line source has attracted much attention. Excitation of plasma waves for a line source parallel to a magnetic field has been analysed (Harris 1963). The line source in a half space (Seshadri 1964) and in a cylindrical cavity (Wait 1964) have also been analysed to ascertain the coupling to electro–acoustic waves at a boundary in a warm plasma. Some attention has also been paid to magnetic line source in an inhomogeneous plasma (Mitchell 1965).

The behaviour of sources of excitation of electromagnetic waves in plasmas near resonant frequency regions and their time dependence requires additional attention because of the contribution due to turbulence, etc, under various conditions. The turbulent motion of the medium makes the medium a dielectrically inhomogeneous medium, and a wave travelling through it and reaching an antenna is subject to amplitude and phase fluctuations. The size of these fluctuations depends on the length of the path in the inhomogeneous medium and the wavelength (the fluctuations increase with the increase of length and with the reduction of wavelength), while the difference between the fluctuations at adjacent points increases with the distance between the points. This is why the role of the turbulence of the medium on antenna operation and parameters increases with the antenna system size and as the wavelength falls. The effect of turbulence on the parameters of an antenna and on its performance have been investigated (Shifrim 1963, Lomakin 1966). Some estimates have been made and they indicate that such regions of operation, in which the effect of fluctuations in the direction of the pattern is a maximum, are insignificant (Stotskiy 1969). In this paper a partial analysis of the radiation from an electric dipole has been made and the expression for the total energy flux of the dispersed field has been obtained. This analysis shows that the turbulence of the medium affects the radiating properties of the antenna.

2. Radiation from an electric dipole in a turbulent plasma

In the propagation of electromagnetic waves in a turbulent medium random phase fluctuations appear. Large scale (compared to the dimensions of the antenna) inhomogeneities of smaller sizes cause relative dephasing of the rays and, hence, to a widening of the main lobe of the radiation pattern of the antenna. The width of the mean radiation pattern of a linear antenna in a turbulent atmosphere has already been calculated (Stotskiy 1969).

To find out the 'intrinsic' width of the mean radiation pattern, the statistical widening of the radiation pattern due to fluctuations is not considered. Therefore, the above definition for the width of the radiation pattern characterizes the resolving power of the radiating source (antenna) in such regimes and the limiting resolution which can be generally achieved in the presence of the turbulent atmosphere.

The equation for the dispersed field in the first approximation is taken in the form:

$$\Delta \boldsymbol{E}_1 - \nabla \operatorname{div} \boldsymbol{E}_1 + k_0^2 \boldsymbol{E}_1 = -k_0^2 \epsilon_1(\boldsymbol{r}) \boldsymbol{E}_0(\boldsymbol{r}) \tag{1}$$

where $\epsilon_p = 1 + \epsilon_1(\boldsymbol{r})$ is the dielectric permeability of the plasma; $\epsilon_1(\boldsymbol{r})$ is a small functional addition; $\langle \epsilon_1 \rangle = 0$; $k_0 = \omega/c$, $\boldsymbol{E}_0(\boldsymbol{r})$ is the unperturbed dipole field in a plasma with no fluctuation. The angular brackets indicate averaging of the random quantity ϵ_1 over an ensemble of samples.

Equation (1) can easily be transformed to the following form:

$$\Delta \boldsymbol{E}_1 + k_0^2 \boldsymbol{E}_1 = -k_0^2 \epsilon_1 \boldsymbol{E}_0 - \nabla(\nabla \cdot \epsilon_1 \boldsymbol{E}_0). \tag{2}$$

Considering the non-uniformities in a finite space, the solution of equation (2) for this space is written as follows:

$$\boldsymbol{E}_1(\boldsymbol{R}) = \frac{k_0^2 \exp(\mathrm{i}k_0 R)}{4\pi R} \int_V \epsilon_1(\boldsymbol{r}) [\boldsymbol{E}_0(\boldsymbol{r}) - \boldsymbol{m}(\boldsymbol{m}\boldsymbol{E}_0(\boldsymbol{r}))] \, \mathrm{d}\boldsymbol{r} \exp(-\mathrm{i}k_0 \boldsymbol{m} \,.\, \boldsymbol{y}). \tag{3}$$

Here, Gauss's theorem has been used for the transformation. The integration has been taken over the volume occupied by the non-uniformities. The dipole under consideration is inside this volume at the origin of the coordinates, $\boldsymbol{m} = \boldsymbol{R}/R$, where $\boldsymbol{R} = R(R, \theta, \phi_0)$ is the radius vector of the observation point. The intensity of the dispersed radiation is given by

$$I\,\mathrm{d}\Omega_I = \frac{c}{8\pi}\langle \boldsymbol{E}_1(\boldsymbol{R})\boldsymbol{E}_1^*(\boldsymbol{R})\rangle R^2\,\mathrm{d}\Omega_I \tag{4}$$

where $\mathrm{d}\Omega_I$ is the solid angle.

Using equation (3) in equation (4) the energy obtained is given as follows:

$$I = \frac{\omega^4\langle\epsilon_1^2\rangle}{2(4\pi c)^3}\int_V\int_V \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\eta_{\epsilon p}(\boldsymbol{r},\boldsymbol{r}')\exp[\mathrm{i}k_0\boldsymbol{m}(\boldsymbol{r}-\boldsymbol{r}')][\boldsymbol{E}_0^*(\boldsymbol{r})\boldsymbol{E}_0(\boldsymbol{r}')-\boldsymbol{m}\boldsymbol{E}_0^*(\boldsymbol{r})\boldsymbol{m}\boldsymbol{E}_0(\boldsymbol{r}')] \tag{5}$$

where $\eta_{\epsilon p}(\boldsymbol{r},\boldsymbol{r}')$ is the correlation coefficient of the fluctuations. If the fluctuations are uniform and isotropic, $\eta_{\epsilon p}(\boldsymbol{r},\boldsymbol{r}') = \eta_{\epsilon p}(|\boldsymbol{\rho}|)$ where $\boldsymbol{\rho} = \boldsymbol{r}-\boldsymbol{r}'$. Since $\eta_{\epsilon p}(\boldsymbol{\rho})$ falls off quickly for $\rho > d_c$. The quantity d_c is a characteristic non-uniformity dimension; $\boldsymbol{E}_0(\boldsymbol{r}') = \boldsymbol{E}_0(\boldsymbol{r}-\boldsymbol{\rho}) \simeq \boldsymbol{E}_0(\boldsymbol{r})$ is assumed for small scale $k_0d_c \ll 1$ which has the correlation function of the form:

$$\eta_{\epsilon p}(\rho) = 4\pi\langle\epsilon_1^2\rangle d_c^3\Delta(\rho). \tag{6}$$

Here

$$d_c^3 = \int_0^\infty \eta_{\epsilon p}(\rho)\rho^2\,\mathrm{d}\rho.$$

Using equation (6) in equation (5), a divergent expression for the energy flux is obtained since equation (6) is not valid in the immediate vicinity of the dipole where the field diminishes markedly at distances much less than λ (the wavelength). It is known that the energy flux of the dispersed field with the spatial distribution of the random non-uniform plasma taken into account is finite. Such a method, however, permits us to obtain only the full energy flux of the dispersed field. To obtain the energy flow density of the dispersed field, however, one must successively compute the finite non-uniformity dimensions, which is equivalent to computing the spatial dispersion of the randomly non-uniform plasma.

Considering $\eta_{\epsilon p}(\rho) = \exp[-(\boldsymbol{r}-\boldsymbol{r}')^2/L^2]$ for the dispersion volume in the form of a sphere with its centre at the origin of the coordinates and a radius of R_0; the equation (5) is transformed to the following equation:

$$I = \frac{\omega^4\langle\epsilon_1^2\rangle}{120\pi c^3}[\tfrac{1}{4}(6+\sin^2\theta_0)+k_0^4d_c^3R_0(2+7\sin^2\theta_0)]. \tag{7}$$

Here, the first term of equation (7) is the contribution of the quasistatic field to I and the second term is the wave contribution. Carrying out the integration of equation (7) over angles, the full energy flux of the dispersed field

$$I_f = \int I\,\mathrm{d}\Omega_I = \frac{2\omega^4\langle\epsilon_1^2\rangle}{9c^3}(\tfrac{1}{4}+k_0^4d_c^3R_0), \tag{8}$$

which is in agreement with the expression obtained by the radiation reaction method.

The problem can be simplified by considering that there are no non-uniformities in the immediate vicinity of the dipole. Then, the dispersion volume can be taken in the form of a spherical layer with an inside radius of r_0; if $r_0 \gg d_c$, equation (6) can be used. This type of presentation of the problem is possible for a radiator with a dimension of $l \gg d_c$. From equation (6) for the spherical layer, the full energy flux of the dispersed field is given as follows:

$$I' = \frac{\omega^4\langle\epsilon_1^2\rangle d_c^3}{60\pi c^3}\left(\frac{1}{r_0^3}(11+6\sin^2\theta_0)-\frac{k_0^2}{4r_0}(3+\cos^2\theta_0)+\frac{k_0^4R_0}{2}(2+7\sin^2\theta_0)\right)$$

and

$$I'_f = \int I'\,\mathrm{d}\Omega_I = \frac{\omega^4\langle\epsilon_1^2\rangle d_c^3}{18c^3}\left(\frac{18}{r_0^3}-\frac{k_0^2}{r_0}+4k_0^4R_0\right), \tag{9}$$

where the first two terms are the results of the dispersion of the immediate-vicinity fields, which made an important contribution, under the condition that $k_0^4R_0r_0^3 \ll 1$, to that obtained above for equations (7) and (6), here the parameter r_0 takes the form of d_c. It may also be seen that as $r_0 \to 0$, I', $I'_f \to \infty$ which shows that equation (6) cannot be used for a point-source radiator ($l \ll d_c$).

3. Results and conclusions

A partial analysis of the radiation from an electric dipole in a turbulent plasma has been made, and the expressions for the intensity of the dispersed field as well as for the total energy flux of the field have been obtained. The analysis shows that the turbulence of the plasma medium affects the radiating properties of the antenna. The intensity of the dispersed radiation and the total energy flux have been calculated (table 1 and 2, respectively). Also, the total energy flux of the dispersed field through the spherical layer when no uniformities are present in the immediate vicinity of the dipole has been calculated (table 3).

In conclusion, the behaviour of the sources of excitation of electromagnetic waves in plasmas near resonance frequency regions and their time dependency require additional

Table 1.

$I = \text{constant} \times A$

$$\text{Constant} = \frac{\omega^4\langle\epsilon_1^2\rangle}{120\pi c^3} = 6{\cdot}25\times10^{-19}$$

	A			
d_c	$\theta_0 = 0°$	$\theta_0 = 30°$	$\theta_0 = 60°$	$\theta_0 = 90°$
0·0	1·5000	1·5625	1·6875	1·7500
0·1	1·5200	1·6000	1·7600	2·8400
0·2	1·6600	1·8625	2·2675	3·4700
0·3	2·1000	2·6875	3·8625	4·4500
0·4	2·7000	3·8125	6·0375	7·1500
0·5	3·5000	5·3125	8·9375	10·7500
0·6	5·5000	9·0600	16·1875	19·7500
0·7	10·5000	18·4375	34·3125	42·2500
0·8	11·5000	20·3125	37·9375	46·7500
0·9	15·5000	27·8125	52·4375	64·7500
1·0	21·5000	39·0625	74·1875	90·7500

Table 2.

$I_f = \text{constant} \times B$

$$\text{Constant} = \frac{2\omega^4\langle\epsilon_1^2\rangle}{9c^3} = 5 \times 10^{-17}$$

Sample number	d_c	B
1	0·0	0·2500
2	0·1	0·2600
3	0·2	0·3300
4	0·3	0·5200
5	0·4	0·8900
6	0·5	1·5000
7	0·6	3·4100
8	0·7	5·0800
9	0·8	5·3700
10	0·9	7·5400
11	1·0	10·2500

Table 3.

$I_f' = \text{constant} \times C$

$$\text{Constant} = \frac{\omega^4\langle\epsilon_1^2\rangle}{60\pi c^3} = 12{\cdot}50 \times 10^{-19}$$

	C			
d_c	$\theta_0 = 0°$	$\theta_0 = 30°$	$\theta_0 = 60°$	$\theta_0 = 90°$
0·1	0·0200	0·0303	0·0509	0·0613
0·2	0·1600	0·2426	0·4075	0·4900
0·3	0·6000	0·9096	1·5282	1·8375
0·4	1·2000	1·8192	3·0564	3·6750
0·5	2·0000	3·0320	5·0940	6·1250
0·6	4·0000	6·0640	10·1880	12·2500
0·7	9·6000	14·5536	24·4512	29·4000
0·8	10·0000	15·1600	25·4700	30·6250
0·9	14·0000	21·2240	45·7481	42·8750
1·0	20·0000	30·3200	50·9400	61·2500

attention because of the contribution due to turbulence under various physical conditions. Here, the effect of turbulence on the radiation of the radiating sources has been discussed partially. The radiating properties of a radiating source surrounded by a turbulent plasma in different situations could be investigated. These have not been performed completely but are presently under study. When these have been completed it would be possible to analyse the radiating properties of a radiating source in a turbulent plasma. The analysis could aim at determining the Poynting vector, the radiation pattern and radiation resistance as a function of plasma parameters as well as of signal. The results would be discussed for the radiation properties produced by antennae under various physical situations.

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